Tipping points among social learners: Tools from varied disciplines

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Abstract   There is a long and rich tradition in the social sciences of using models of collective behavior in animals as jumping-off points for the study of human behavior, including collective human behavior. Here, we come at the problem in a slightly different fashion. We ask whether models of collective human behavior have anything to offer those who study animal behavior. Our brief example of tipping points, a model first developed in the physical sciences and later used in the social sciences, suggests that the analysis of human collective behavior does indeed have considerable to offer [*Current Zoology* 58 (2): 298−306, 2012].

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Animal behavior is one arena where social learning, defined as learning by observing or interacting with others (Heyes, 1994) as opposed to learning individually, is studied in its most uncomplicated form (e.g., Laland, 2004; Couzin et al., 2005). Nevertheless, groups of animals in which individuals learn from and respond to other individuals are still complex systems. For example, in flocks of birds or schools of fish, it takes only a small fraction of individual learners to impart a coherent direction to the entire group, as the majority copies the travel direction of neighbors (Couzin et al., 2005). Flocks gain their coherent directionality through averaging the individual perceptions of each bird.

By studying the collective behavior of animals, the dynamics of these systems can be observed and manipulated clearly enough to often lend insight into the core of much more complex dynamics of human societies and groups (Laland and Reader, 2010). The relative simplicity of rules by which animals interact can help us see through the fog of complicated factors that pervade human social behavior. What about the reverse? Can studies of collective human behavior tell us anything important about animal behavior?

As an example we focus on a single issue, tipping points in human (and mechanical) systems. Although its origin lies in the sociology of the 1950s, the term "tipping point" has risen dramatically in popularity only in the last decade, catalyzed by Gladwell's (2000) best-selling book, and now regularly used in studies of such topics as climate change (Russill and Nyssa, 2009) and finance (Scheffer et al., 2009).

1   Scaling Up

Although care needs to be taken in scaling up observations on flocks of birds or schools of fish to human groups, they provide potentially profound hypotheses about collective decisions in human groups, in that the most persuasive or influential property may not necessarily be logical superiority in skill or status—increased working memory, ability to delay gratification, and the like—but rather the persistence of the message, a strategy of which opinion leaders and marketers make explicit use. This effect may arise from the network rather than from a special individual. Highly clustered social networks, for example, appear to favor the spread both of norms of cooperation (Ohtsuki et al., 2006) and of innovations by introducing them repeatedly to individuals through different neighbors of a cluster (Helbing and Yu, 2009; Centola, 2010; Lorenz et al., 2011).

Studies of collective animal behavior can also tell us something about the so-called "wisdom of crowds" (Suroweicki, 2004). A group of animals migrates long distances through lots of "noisy" directional cues that are smoothed out and integrated across the group, through social learning among individuals. Similarly, experiments in psychology have shown that a group of people with average intelligence can brainstorm better and plan their activities more effectively than "smarter" individuals (Wooley et al., 2010).

Studies of animal behavior not only provide data

relative to the averaged behavior of large groups, they also highlight the presence of abrupt changes. For flocks of birds, for example, false alarms can spread and amplify across a flock through bad information (Couzin et al., 2005). Hence, in both human and animal groups, the element of individual thinking and accurate information is critical (Bentley and O'Brien, 2011). Even simple forms of animal social learning give a group higher-order computational capacities to respond to its environment (Couzin, 2007).

2 Scaling Down

Given the leap in cognitive ability that humans possess, are human models applicable to the study of animals? Multiple animal species are able to learn (Laland and Reader, 2010), but only groups of humans—more accurate and complex social imitators than any other animals—can substantially accumulate socially learned information over generations. Humans continue to "learn things from others, improve those things, transmit them to the next generation, where they are improved again, and so on," and this process continues to lead to the "rapid *cultural* evolution of superbly designed adaptations to particular environments" (Boyd and Richerson, 2005: 4, emphasis in original). Human cultural transmission is thus characterized by the so-called "ratchet effect," in which modifications and improvements stay in the population until further changes ratchet things up again (Tomasello et al., 1993; Tennie et al., 2009).

The question is, if collective human behavior is so complex that it far outstrips the processes and components that scaffold animal behavior, perhaps the application of the former to the later is analytically naive. We reject this proposition on two grounds. First, a vast amount of research has shown that the collective behavior of many animals—birds, ants, locusts, Morman beetles, and hundreds of other taxa (e.g., Couzin et al., 2005; Buhl et al., 2006; Bode et al., 2010; Eriksson et al., 2010; Katz et al., 2011)—are as varied and as complicated as any collective behaviors exhibited by humans. Second, and perhaps counterintuitively, many models of collective human behavior—stock-market activity, for example—make undemanding assumptions about human intelligence and in fact have been termed "zero-intelligence" models (Farmer et al., 2005). Although zero-intelligence models in finance are concerned with how traders arrive at prices for equities, the central mechanism is the same as the new breed of models of flocking and herding: agents copy the actions of other agents with a certain degree of error. This combination of transmission and error should sound familiar, as it reflects two of the three elements of the evolutionary process, inheritance and variation. This is why these simple models are so powerful.

It appears that sociological models may have informed zoological models in terms of how a consensus or preference forms. Palfrey and Poole (1987), for example, modeled "voters" as varying in how well informed they were about political candidates. Starting with a bimodal distribution of voter preference, they found that adding uninformed agents increased the tendency for the consensus to regress to a single mode. Uninformed individuals effectively diluted the influence of those who were better informed. Similarly, Couzin and colleagues (2011) considered the role of neutrality, or ignorance, among group members in achieving group consensus. As they show, it is possible for group consensus to be controlled by a small but determined minority that has much stronger preferences than the majority. In addition, the transition from the majority-rules state to the minority-rules state occurs in tipping-point fashion. A rapid transition occurs between dynamic phases as the strength of preferences in the informed minority is increased and/or the fraction of uninformed members of the population is increased (to a point).

Using as an analogy flocks or schools, each agent is assigned a direction vector, and the strength of its preference (directionality) is captured by the magnitude of that vector. The only other rules are that each agent aligns with the direction of travel of its neighbors and avoids collisions. Couzin and colleagues supposed there are two subpopulations—a majority traveling by following direction vector w_1 and a minority following direction vector w_2. The group normally follows the majority direction except as w_2 is made greater than w_1, whereupon the group suddenly directs itself in the minority direction (Fig. 1a). These results were nicely confirmed by controlled experiments with fish (golden shiners *Notemigonus crysoleucas*) trained to prefer a blue or a yellow target destination and then released in different proportions from the tank side opposite those target destinations (Couzin et al., 2011).

A second version of Couzin and colleagues' model is a binary-choice model that is effectively equivalent to the socio-ecological models of Watts (2002) and Haldane and May (2010), which we will describe in the next section. Couzin and colleagues set agents within a fixed network and allowed them to adopt, probabilistically, the choice they perceived to be the majority in
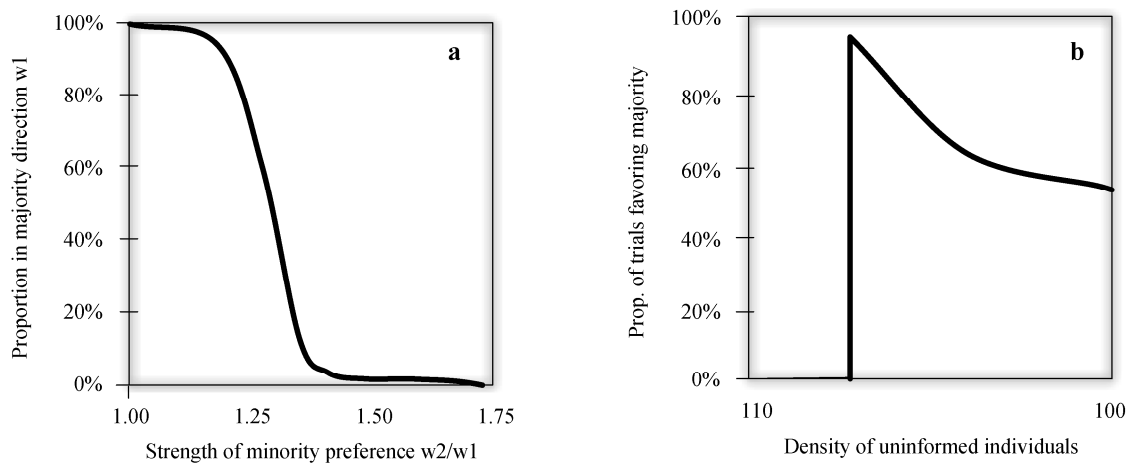
Fig. 1   Tipping points in the collective behavior model of Couzin et al. (2011)
As the directional preference of the minority (w_2) is made greater than the preference of the majority (w_1), the group suddenly directs itself in the minority direction (left-handed). As the density of uninformed individuals is increased, the consensus abruptly shifts from minority to majority (right-handed).

their local network. Each agent also had its own level of "intransigence" during interactions, equivalent to the social-conformity thresholds explored by Watts (2002). The result was again a sharp tipping point from minority- to majority-controlled outcome. As Fig. 1b shows, as the density of uninformed individuals is increased, the consensus abruptly shifts from minority to majority, as uninformed individuals inhibit the influence from the strongly opinionated minority (Couzin et al. 2011). This resembles tipping points studied in social sciences.

3   Tipping Points

Prior to the study of Couzin and colleagues, tipping points in group dynamics have also been observed and modeled in the behaviors and interactions of social insects, for example (Beekman et al. 2001; Theraulaz et al. 2002; Amé et al. 2006). Perhaps because language affects how science is practiced (Griffiths and Steyvers, 2004; Evans and Foster, 2011), there is a resurgence in the academic study of tipping points, especially as they relate to the social sciences. The value of this body of research for animal scientists lies in the complex-systems approach—a value that is increasingly being realized through studies that treat human systems and ecological systems interchangeably (e.g., Dyer et al., 2009; Saavedra et al., 2009; Haldane and May, 2011). Generally speaking, the fundamental question is how, through a change in one system parameter, an abrupt change occurs in another measure that describes the state of the system. Parameters and changes can take many forms, and in some cases it is easy to see the potential link to collective animal systems. Schelling (1971), for example, ostensibly was concerned with

ethnic segregation, but the crucial parameter in his model was the threshold that modeled agents had for different neighbors, the critical value of which segregated the modeled population.

Another classic tipping-point parameter is the average number of network connections per agent at which a random network becomes a giant interconnected cluster (Erdős and Rényi, 1960). Similarly, a tipping point might describe a bifurcation in the behavior of an iterated equation (Brock and Hommes, 1997), such as the classic logistic map $x_{t+1} = Ax_t(1-x_t)$, which, as A is increased from 3.0 to 4.0, becomes chaotic, such that each result is never repeated in subsequent iterations.

Although highly instructive, rarely are the parameters of these models measurable or realistic enough for tipping points to be *predicted* in the world of interacting animals. Typically what are measured are the characteristics of a time series of a single system variable (e.g., Scheffer et al., 2009) or the coordination of nontemporal factors such as the interconnectedness of the system and the propensity for each of its individual agents to move or change (e.g., Haldane and May, 2011). Given the cost-benefit limits on predictability (Goldstein and Seheult, 2008), such measurable characteristics can be subject to certain simple tools for evaluating tipping points, through tests of quantitative time-series data and qualitative interaction. These are termed "threshold assessments."

4   Time Series: Simple Tricks

The record of the past is our best view into the future, and in the science of behavior we are often presented with a time series of events, from abundances of fossil

species to climatic events or the incidence of certain animal behavior observed in the lab or the field (e.g., Ormerod, 1998, 2006; Michie, 2006; Long, 2009; Willis et al., 2009). Often faced with measurements of one state variable—algae population, stock price, observed incidences of an animal behavior—plotted over time, researchers are often motivated to squeeze as much out of these time series as possible. Recently, Scheffer et al. (2009) reviewed the tools applicable to time-series data in identifying signals that might predate a tipping point. In their review, tipping point refers to the point in time when a system (a) jumps abruptly to a new steady state, (b) bifurcates into a pattern of oscillation between attractor states, or (c) transitions into chaotic behavior (Scheffer et al. 2009; see also Lamberson and Page, 2012). For any given time series of events, the critical tools that Scheffer et al. (2009) identify include

1. A critical slowing down, where the response time to an external perturbation takes longer and longer to return to the steady state, the tipping point being when it ultimately never returns;

2. An increasing range and significance of the autocorrelation within the time series, the simplest form of which is the degree to which the state at time t_{-1} predicts the state at time t;

3. A skewness in the response, such that the magnitude and response time in one direction (increase or decrease in the parameter describing the system) is much longer than those in the other direction. This signals that the basin of attraction is asymmetrical and that the system may soon "roll off" in the direction away from the current attractor; and

4. A flickering between discrete values of the system parameter, signaling that the system may already have reached bifurcation.

In principle, these criteria can be applied to time series representing a large range of systems of interacting elements.

Before considering animal systems, it might be helpful to first consider how these tools were used for analyzing other systems. Dakos and colleagues (2008), for example, showed how a critical slowing down in time series of climate-proxy data preceded eight different abrupt climate changes in the Earth's history. In ecology, a time series of population size was shown to foreshadow a critical transition in extinction equilibrium through pronounced autocorrelation, increased skewness, and a critical slowing down (Drake and Griffen, 2010). The human system most often compared metaphorically to an animal herd is the stock market. Aside

from the rough analogy, tools used for financial data have successfully been applied to animal-behavior data. One example concerned the data on animal group size collected by Hill and Dunbar (2003), which struck the interest of researchers who analyze financial time series in detail. The study (Zhou et al., 2005) revealed the hierarchical nature of animal social groups in that the group-size data showed a modularity such that each new order of group size was three times larger than the previous one. Those animals happened to be humans, and the data concerned their networks of holiday cards sent to each other (Hill and Dunbar, 2003), but the insight about the hidden hierarchical organization then led, by way of a similar discovery about hunter–gatherers (Hamilton et al., 2007), to essentially the same realization about hierarchical groupings of baboons, orcas, and elephants (Hill et al., 2008). In other words, the social study of humans, combined with insights from a geophysicist and an expert on financial time series, produced new insights into animal behavior.

5 System Interconnections and Thresholds

Ideally, time-series analysis would be complemented by a quantitative, or even a qualitative, assessment of the configuration of the different parts and their interactions. Network science is an increasingly popular approach, but for many dynamical systems the actual interconnections, especially brief encounters between animals, are ephemeral or so numerous as to be uncountable. In comparison, an ecosystem model may offer significant new insights into the dynamics, particularly where uncertainty is high. For example, after demonstrating that climate change would disrupt entire networks of bird populations across Sub-Saharan Africa, Hole and colleagues (2009) advocated increasing the number and size of protected areas, restoring critical habitat types, and active monitoring. These recommendations resemble those offered for business in an uncertain environment, which include increasing the number of bets made in the market and continual monitoring and feedback (Watts and Hasker, 2006).

Bringing insights from ecology into finance, Haldane and May (2011) used a well-researched ecosystem model that assumes there exists a population of discrete, interacting agents—species, people, perhaps social animals—and the individual elements of the system are characterized by some threshold to movement. Haldane and May just happened to be interested in banks rather than in any of these zoological phenomena—their

"ecosystem" featured banks as the species, interbank loans as their interconnections, and cash reserves as the threshold to a bank's extinction. This system was prone to mass extinctions. If a bank's cash reserves fell below zero, then it failed, which meant that any banks dependent on it for interbank loans were suddenly cut off, such that they might fail, and so on. Clearly, the closer to the bone that the banks operated, with just a sliver of cash assets above their liabilities, the higher the risk of failure and the more prone the modeled system was to catastrophic failure.

The general lesson here is to consider systems in terms of interconnections and thresholds to change rather than in terms of the details of individual agents. In other words, interconnections and thresholds are what should be studied more intensively in terms of the distribution of each measure per agent. Do all agents have the same number of connections and the same threshold to change, or are they distributed normally or in a non-Gaussian manner? Do the distributions change over time? These kinds of information are labor-intensive to collect in the field, in terms of observing animal interactions, but the recognition of the kinds of data needed could lead to a radical change in collection strategies.

Haldane and May's (2011) model also reveals, less intuitively, that the degree of interconnection between agents also affects the resilience of a system. This can be seen in a plot of the space of possibilities, with the average threshold on the x-axis and the average number of interconnections per agent (assumed to be normally distributed) on the y-axis. In Fig. 2a, the shaded area indicates the space in which the banking system of Haldane and May is prone to system-sweeping ava-

lanches of change. One can see that lowering the threshold brings the system into this region, where there is a middle range of interconnectivity (the point at which the shaded region is fattest).

In the abstract, this space of potential cascades, which is dependent on interconnections versus individual thresholds to change, is the same as that yielded by a model of information cascades across a network of social agents. Watts (2002) modeled social agents as being capable of a binary decision—adopt or not adopt a behavior. We know that different behaviors diffuse through chimpanzee groups, for example (Whiten and Mesoudi, 2008), so this kind of analysis may be applicable to the recognition of chimpanzee "cultures" across geographic regions (Whiten et al., 1999).

Watts's (2002) model was based on a classic experiment in social conformity (Asch, 1955), where each agent in the network had a threshold of the fraction of neighboring agents (in a random network with mean of k connections per agent) that had to adopt an idea before he would adopt it. Each time an agent switched, this changed the friends' circle for its surrounding agents and might cause one of them to switch, and so on, percolating through the "vulnerable cluster" of agents (Watts, 2002). In running the model over the range of parameter values, Watts produced a map of cascade potential (Fig. 2b) that was quite similar to the model of bank-failure cascades presented by Haldane and May (2011).

These ecosystem models, and their generality, are closely related to a family of models from the 1990s that considered evolution as being analogous to highly interconnected physical systems (sand piles and forest
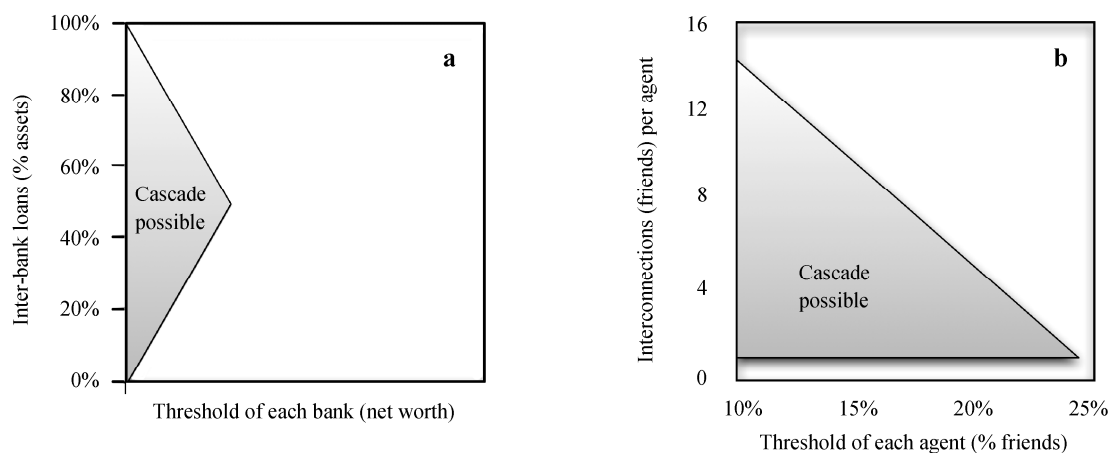

Fig. 2   **The model of Haldane and May (2011), showing the combination of factors (average net worth of banks versus interconnections per bank) where the "ecosystem" of banks is susceptible to a sweeping cascade of failure (left-handed). The model of Watts (2002), showing the same region of susceptibility of cascades, but in this case the tipping point leads to an innovation sweeping across a social network (right-handed).**

fires were common models), where small changes could, once in a while, trigger massive avalanches of change. Both could be described, in the words of the day, as "self-organized critical" systems (Bak et al., 1987), precariously balanced between the building up of inter-dependencies. As with the addition of a grain of sand to the sand pile, the extinction of a single species could cause the extinction of other species that were ecologically dependent on it (Sneppen et al., 1995; Paczuski et al., 1996; Solé and Manubria, 1996).

As often happens in science, one aspect of the thinking gets selected and others get left behind. We think it is worthwhile to revisit the thinking of Stuart Kauffman (1993, 1995), who in his now-classic modeling of a system of interacting agents revitalized Sewall Wright's 1930s concept of the fitness landscape undergoing constant change, which Kauffman called "dynamic" fitness landscapes. Kauffman showed that if agents are connected only moderately, then by adjusting their strategies accordingly, agents can all adapt fairly easily. However, as things become more interconnected, agents must choose either to make small adjustments to optimize their current strategy or undertake major, risky changes in order to seek a better long-term strategy. When the network is completely interconnected, highly favorable strategies may be impossible to locate. This "hill-climbing" approach has now been effective in understanding how animals as cognitively limited as sticklebacks adapt collectively (e.g., Kendal et al.,

2009).

6 Distributions

One signature of Kauffman's model was a highly right-skewed distribution in the range of possible chain reactions, from the majority of very small changes to the minority of catastrophically large avalanches. In the 1990s, distributions were at the center of a debate about the evolutionary process that creates distributions of cascade sizes and durations (e.g., Sneppen et al., 1995; Newman, 1996, 1997; Paczuski et al., 1996; Solé and Manubria, 1996; Kirchner and Weil, 1998; Newman and Eble, 1999a, 1999b). Different explanations for the distributions provide multiple testable hypotheses that can be compared against real data, such as the distribution of branch lengths of a phylogenetic tree (Burlando, 1990; Bak and Boettcher, 1997). By comparing a range of different distributions (exponential, Weibull, log-normal, almost power law), Venditti et al. (2010) discovered that the branch lengths of fossil evolutionary trees are often distributed by an exponential function.

The new complexity science highlighted the question of whether events in a time series (such as a fossil record) were caused independently, by extreme external events, or possibly through the vast tangle of interconnections, such that a small event could trigger a cascade. Making Kauffman's model even more simple, Sneppen et al. (1995) modeled their "ecosystem" as the arrangement of index numbers ("species") in a circle, such that
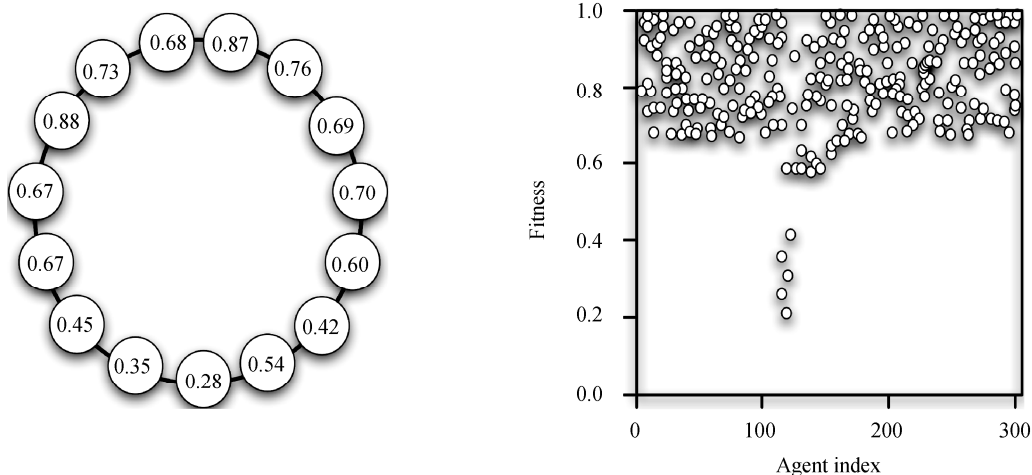

Fig. 3 In the Sneppen et al. (1995) model (after Paczuski et al., 1996), as shown at left, agents are arranged in a circle and assigned fitness ("barrier") values between 0 and 1. Then, at each time step in the simulation, the smallest fitness, together with the fitness of the two neighbors, are each replaced with new random fitness values. This step is repeated again and again. On the right is a "snapshot" of agent fitness during the critical state. The x-axis shows the agents' positions on the circle, and the y-axis shows their fitness at a single time step. Note that almost all agents have fitness values above a self-organized threshold of 0.667, but a coevolutionary avalanche is occurring among the species at around $x = 120$.

each agent interacts with its two nearest neighbors (Fig. 3). Each agent is represented by a fitness value between 0 and 1, chosen at random to start the simulation. At each time step of the simulation, the lowest fitness value in the circle is selected against and assigned a new random value. The two nearest neighbors of the chosen agent are also randomly assigned new fitness values, as the "adaptation" of dependent agents.

The dynamics of this simple model, however, are surprisingly rich and complicated (e.g., Paczuski et al., 1996). In essence, the model holds the essential elements of inheritance and variation we have described so far: through selection against the worst fitness in each time step, the fitnesses of all the remaining agents rise over time, but then the "ecosystem" inevitably reaches a tipping point. All the species are fit, meaning that the next ones taken out are replaced by random values, such that one of these new agents is likely chosen again, and so on, as a localized avalanche forms, with a growing cluster of random fitnesses within a larger world of fitnesses above the critical value. Just as in the Haldane and May (2011) model, avalanches can be made more likely by making agents more densely interconnected (Sneppen et al., 1995; Paczuski et al., 1996).

All these models have the common element of thresholds and interconnections. Later, Newman (1997) presented a simple model with no interconnections, from which he was able to produce similar power-law distributions, and Newman (2005) has since showed convincingly that multiple processes can generate these distributions. This work was highly productive and needs to be investigated by those currently working on tipping points so that we do not end up reinventing the wheel. For example, recent studies of social networks often present bidirectional links between agents of only one kind, and yet 15 years ago, when Solé and Manubria (1996) varied the Sneppen et al. (1995) model by assigning variable strengths and signs to agent interconnections (they can be beneficial or harmful relationships), they produced rich new dynamics. These need to be revisited by tipping-point enthusiasts.

In sum, when we look back at the last 20 years of modeling tipping points in collective behavior, we see that many of these models focus on the intricacies of the agents within the group only in terms of (a) what the thresholds are to movement or change and (b) how they are interconnected with each other. Among the measurable emergent phenomena from these systems, two of the most highly studied in human systems have been time series of the frequency in a certain behavior and

distributions of frequency among all the different possible behavior within any given time period.

7   Applications to Current Zoology

Analytical emphasis on models of human collective behavior could justify substantial changes in how information is collected in zoological fieldwork, particularly for social animals. A behavior can thus be observed among individuals but also depicted among a network of individuals. In animal social-diffusion experiments (e.g., Whiten and Mesoudi, 2008), is the assumption that the behavior can be copied between any two individuals, or is there a network involved? And if the network is not practically observable, can the emergent patterns such as the distributions of cascade sizes perhaps be used to infer how events were interconnected (Venditti et al., 2010)? Can certain groups be characterized as nearer to a tipping point based on observations of network density, time series of behavioral events, distribution of frequency of behaviors, or a sustained increase in the variance of a behavior (Carpenter and Brock, 2006)? These questions could be explored through new ways of collecting and thinking about data on social animal behavior.

We therefore suggest three tools might be brought to bear: 1. Simple time series analysis as advocated by Scheffer et al. (2009); 2. Study of distributions resurrected by Venditti et al. (2010); and 3. Explicit consideration of interconnections and thresholds: may require different field information-collection strategies.

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